Exercise 3

Use established properties of moduli to show that when $|z_3| \neq |z_4|$,

$$\frac{\operatorname{Re}(z_1 + z_2)}{|z_3 + z_4|} \leq \frac{|z_1| + |z_2|}{||z_3| - |z_4||}.$$

Solution

Inequality (3) in the text states that for a complex number z,

$$\operatorname{Re} z \leq |\operatorname{Re} z| \leq |z|. \tag{3}$$

Inequality (8) in the text states that for two complex numbers, z_1 and z_2,

$$|z_1 \pm z_2| \geq ||z_1| - |z_2||. \tag{8}$$

Use inequality (8) to make the denominator smaller and inequality (3) to make the numerator bigger (the fraction becomes bigger as a result in both cases).

$$\frac{\operatorname{Re}(z_1 + z_2)}{|z_3 + z_4|} \leq \frac{|z_1 + z_2|}{||z_3| - |z_4||}$$

Apply the triangle inequality to make the numerator even bigger.

$$\leq \frac{|z_1| + |z_2|}{||z_3| - |z_4||}$$